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# Input and output strictly passive $H_\infty$ control of continuous switched singular systems

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**Abstract**
In this work, the problem of input and output strictly passive $H_\infty$ control is investigated, respectively, by proportional plus derivative state feedback (PDSF) and derivative state feedback (DSF). First, the concept of quadratically normal and quadratically stable (QNQS) is extended to switched singular systems. Second, some sufficient conditions are obtained to guarantee that the closed-loop switched singular systems are QNQS and satisfy $H_\infty$ performance, and explicit expressions of PDSF and DSF controllers are given. Finally, simulation examples are given to show the effectiveness of the proposed methods.

**Keywords** Switched singular systems · Input and output strict passivity · $H_\infty$ control · Proportional plus derivative state feedback

**Mathematics Subject Classification** 93-11

## 1 Introduction

In this work, we aim to study the issue of stabilization and $H_\infty$ performance analysis for continuous switched singular systems. Our motivations mainly come from the following aspects. (1) State jumps of switched singular systems are almost unavoidable at switching instants because of incompatible initial conditions. It is difficult to eliminate state jumps using

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simple proportional state feedback (PSF). Many results on stability were achieved under an assumption that states do not jump at switching instants (see Gao et al. 2015; Yang et al. 2014; Zamani and Shafiee 2014; Zamani et al. 2015). (2) Impulse behavior often occurs in some singular subsystems, which may cause the whole system to be unstable. Some results about such systems were obtained under an assumption that each subsystem is regular and impulse-free see (Lian et al. 2017; Wang et al. 2015; Zamani et al. 2013; Zhou et al. 2013). How to eliminate impulse behavior and state jumps is a meaningful subject. (3) The passivity links the Lyapunov stability and the $L_2$ stability well and provides a powerful tool for control system analysis (Gao et al. 2018), and to the best of the authors' knowledge, no attempt has been made to study the problem of $H_\infty$ control based on input and output strict passivity for switched singular systems.

From the above three aspects, it is of great significance to study the issue of input and output strictly passive $H_\infty$ control by using PDSF and DSF. In Zhai et al. (2016), the problem of robust $H_\infty$ control for continuous switched singular systems with uncertainties in the state, input, and derivative matrices was studied by using an improved average dwell time approach. A delay-dependent linear matrix inequality condition was obtained which can guarantee that the uncertain singular time-delay systems subject to actuator saturation are robustly exponential admissible and satisfy $H_\infty$ performance in Fu and Ma (2016). In Ma et al. (2015), the problem of finite-time $H_\infty$ control for a class of discrete-time switched singular time-delay with actuator saturation was studied using the average dwell-time approach. The finite-time $H_\infty$ control problem for singular stochastic systems with actuator saturation and time-varying state delay was studied through the sliding-mode approach in Ma et al. (2018). The finite-time $H_\infty$ static output-feedback control problem for a class of discrete-time switched singular time-delay systems subject to actuator saturation was investigated by using the multiple Lyapunov-like function method and the average dwell-time concept of switching signal in Ma and Fu (2016). The $H_\infty$ control problem for a class of uncertain switched nonlinear singular systems with time delay was studied using a Lyapunov–Metzler inequality approach, and an observer-based output feedback $H_\infty$ controller was given in Zhao et al. (2013). The problem of $H_\infty$ control was studied for switched singular systems using output feedback control in Shi (2013). The PSF controllers were designed in Fu and Ma (2016), Ma et al. (2015), Wang et al. (2015), Yang et al. (2014), Zhai et al. (2016) and output feedback controllers were used in Ma and Fu (2016), Shi (2013), Zhao et al. (2013). It is well known that impulse behavior of some singular subsystems cannot be eliminated only by PSF control. Many studies indicated that PDSF is an effective way for eliminating impulse behavior of singular systems (see Li et al. 2017; Moulay and Perruquetti 2005; Duan and Patton 1997). In addition, PDSF may eliminate state jumps caused mainly by incompatible initial conditions at switching instants because of the changes in system structure.

Until now, many research achievements on passivity of switched systems have been obtained (see Gao et al. 2018; Li and Zhao 2016; Wu et al. 2013a; Zhao and Hill 2006, 2008). In Gao et al. (2018), $H_\infty$ control of continuous switched systems was studied based on input and output strict passivity. In Wu et al. (2013a), $H_\infty$ control of continuous switched systems was studied based on output strict passivity. Due to the complexity of switched singular system model, a few research achievements on passivity of switched singular systems have been reported (see Lin et al. 2013; Shi et al. 2015; Wu et al. 2013b; Yang and Chen 2018). In Lin et al. (2013), the problem of reliable dissipative control for a class of discrete-time switched singular systems with mixed time delays and multiple actuator failures was studied via the Lyapunov function approach and the average dwell-time scheme. In Shi et al. (2015), passive control of switched singular systems was studied via static output feedback. In Wu et al. (2013b), stability, dissipativity, and passivity-based control of a class of hybrid impulsive switching systems with singular structures and uncertainties were investigated by using switching Lyapunov functions. By generalized Lyapunov function and linear matrix inequality, the issue of robust passive control was studied for uncertain switched singular systems with multiple time-delays in Yang and Chen (2018). Although some related problems of passivity of switched singular systems were studied in Wu et al. (2013b), Lin et al. (2013), Shi et al. (2015) and Yang and Chen (2018), state jumps at switching instants and impulsive behavior were not considered, which may lead to system instability or even collapse.

For the reasons discussed above, the $H_\infty$ control problem based on input and output strict passivity is studied, respectively, for continuous switched singular systems via PDSF and DSF. The researches in Gao et al. (2018) and Wu et al. (2013a) can be regarded as special cases of this work. The main contributions are as follows: (i) input and output strict passivity and PDSF are introduced to study $H_\infty$ control of continuous switched singular systems; (ii) sufficient conditions are obtained such that the closed-loop systems are QNQS and satisfy a prescribed $H_\infty$ norm bound, and PDSF and DSF controllers of singular subsystems with explicit expressions are designed in different cases, respectively; (iii) under the action of the designed controllers and switching laws, impulsive behavior can be effectively eliminated, and state jumps caused by incompatible initial conditions at switching instants can also be eliminated.

## 2 Problem formulation

Consider the continuous switched singular system given by

$$\begin{cases} E_{\sigma(t)}\dot{x} = A_{\sigma(t)}x + B_{\sigma(t)}u_{\sigma(t)} + F_{\sigma(t)}w, \\ z = C_{\sigma(t)}x + G_{\sigma(t)}w, \end{cases} \tag{1}$$

where $\sigma(t) : \mathbf{R}^+ \to \mathrm{M} = \{1, 2, \ldots, m\}$ is the switching signal, and $\sigma(t) = i$ denotes that the $i$th is activated at time $t$. $x \in \mathbf{R}^n$ is the state variable, $u_i \in \mathrm{R}^p$ is the control input, $w \in \mathbf{R}^q$ is the exogenous disturbance, $z \in \mathbf{R}^q$ is the controlled output. $E_i$, $A_i$, $B_i$, $F_i$, $C_i$ and $G_i$ are known real constant matrices with appropriate dimensions, and $\mathrm{rank}(E_i) = r_i \le n$.

**Assumption 1** In this work, we assume that $\mathrm{rank}[E_i \ B_i] = n$ for system (1), $\forall i \in \mathrm{M}$.

**Definition 1** The system $E_i\dot{x} = A_i x$ is said to be QNQS (see Li et al. 2017), if $E_i$ is invertible and there exists a matrix $P_i > 0$ such that

$$P_i(E_i^{-1}A_i) + (E_i^{-1}A_i)^{\mathrm{T}}P_i < 0. \tag{2}$$

According to Definition 1, we extend the definition of QNQS to switched systems as follows.

**Definition 2** The switched singular system

$$E_{\sigma(t)}\dot{x} = A_{\sigma(t)}x \tag{3}$$

is said to be QNQS if derivative matrix $E_{\sigma(t)}$ is invertible and there exists a switching law generated by $\sigma(t)$ such that the whole system is asymptotically stable.

**Definition 3** System $\dot{x} = f(x, w),\ z = h(x, w)$ is said to be input and output strictly passive (see Jiao and Guan 2008), if there exists continuous differentiable positive semi-definite function $V(x)$, such that

$$\dot{V}(x) \le z^{\mathrm{T}}w - \epsilon w^{\mathrm{T}}w - \delta z^{\mathrm{T}}z \tag{4}$$

holds, where $\epsilon > 0$ and $\delta > 0$ are two constants.

The $H_\infty$ control problem based on input and output strict passivity of the system in (1) via PDSF (abbreviated as HCIP) is stated as follows.

**Problem HCIP.** Establish a sufficient condition such that the following conditions hold:

1. System (1) controlled by

$$u_{\sigma(t)} = -K_{\sigma(t)e}\dot{x} + K_{\sigma(t)a}x \tag{5}$$

with $w = 0$ is QNQS.

2. For any $T > 0$ and a given $\gamma > 0$, when $x(0) = 0$, the following inequality holds

$$J_{\mathrm{T}} = \int_0^T (\| z \|_2^2 - \gamma^2 \| w \|_2^2)\mathrm{d}t < 0. \tag{6}$$

## 3 Main results

Initially, we study the problem of $H_\infty$ control for system (1) based on input and output strict passivity when $u_{\sigma(t)} = 0$, and give the following result.

**Theorem 1** *Consider system (1) with $u_i = 0$, and assume that $E_i$ is invertible, if there exist matrices $P_i > 0$ and scalars $\lambda_{ij} < 0$, $(i, j \in \mathrm{M}, i \neq j)$, such that*

$$\begin{bmatrix} \Delta_{i11} & P_i E_i^{-1} F_i - 0.5C_i^{\mathrm{T}} + \delta_i C_i^{\mathrm{T}} G_i \\ * & \epsilon_i I + \delta_i G_i^{\mathrm{T}} G_i - 0.5G_i - 0.5G_i^{\mathrm{T}} \end{bmatrix} < 0, \tag{7}$$

*where $\Delta_{i11} = P_i E_i^{-1} A_i + A_i^{\mathrm{T}} E_i^{-\mathrm{T}} P_i + \delta_i C_i^{\mathrm{T}} C_i + \sum_{j=1}^m \lambda_{ij}(P_j - P_i)$.*

*The switching law is designed as*

$$\sigma(t) = \min\{\arg \max_{i\in\mathrm{M}} x^{\mathrm{T}} P_i x\}. \tag{8}$$

*Then system (1) is QNQS and satisfies $H_\infty$ performance $\gamma = \max_{i\in\mathrm{M}} \sqrt{|1 - 2\delta_i\epsilon_i|}/\delta_i$.*

***Proof*** When $u_i = 0$ and $E_i$ is invertible, system (1) can be transformed into a switched normal system as

$$\begin{cases} \dot{x} = E_{\sigma(t)}^{-1} A_{\sigma(t)} x + E_{\sigma(t)}^{-1} F_{\sigma(t)} w, \\ z = C_{\sigma(t)} x + G_{\sigma(t)} w. \end{cases} \tag{9}$$

Choose multiple Lyapunov functions expressed by $V_{\sigma(t)}(x) = x^{\mathrm{T}} P_{\sigma(t)} x$.

According to the switching law in (8), we define

$$\Omega_i = \{x \in R^n \setminus \{0\} \mid x^{\mathrm{T}}(P_j - P_i)x \leq 0, \forall j \in \mathbf{M}\}. \tag{10}$$

From (10), let

$$\bar{\Omega}_1 = \Omega_1, \ldots, \bar{\Omega}_i = \Omega_i - \bigcup_{j=1}^{i-1} \Omega_j, \ldots, \bar{\Omega}_m = \Omega_m - \bigcup_{j=1}^{m-1} \Omega_j, \tag{11}$$

obviously, $\bigcup_{i=1}^m \Omega_i = \mathrm{R}^n \setminus \{0\}$, $\bigcup\limits_{j=1}^{m} \bar{\Omega}_j = \mathrm{R}^n \setminus \{0\}$, and $\bar{\Omega}_i \cap \bar{\Omega}_j = \phi$, $(i \neq j)$.

When $x \in \bar{\Omega}_i$, the $i$th subsystem of system (9) is activated, and the time-derivative of $V_i(x)$ along the solution of system (9) gives

$$\dot{V}_i(x) = x^{\mathrm{T}}(P_i E_i^{-1} A_i + A_i^{\mathrm{T}} E_i^{-\mathrm{T}} P_i)x + x^{\mathrm{T}} P_i E_i^{-1} F_i w + w^{\mathrm{T}} F_i^{\mathrm{T}} E_i^{-\mathrm{T}} P_i x. \tag{12}$$

From (7) and (12), we can obtain

$$\dot{V}_i(x) < x^{\mathrm{T}}[-\delta_i C_i^{\mathrm{T}} C_i - \textstyle\sum_{j=1}^{m} \lambda_{ij}(P_j - P_i)]x + x^{\mathrm{T}} P_i E_i^{-1} F_i w + w^{\mathrm{T}} F_i^{\mathrm{T}} E_i^{-\mathrm{T}} P_i x. \quad (13)$$

When $w = 0$, from (8) and (13), we can get $\dot{V}_i(x) < -\delta_i x^{\mathrm{T}} C_i^{\mathrm{T}} C_i x \leq 0$. According to the multiple Lyapunov functions theory and Definition 2, we have thus proved that system (1) with $u_i = 0$ is QNQS.

From (7)–(9) and (12), we have

$$\dot{V}_i(x) < z^{\mathrm{T}} w - \epsilon_i w^{\mathrm{T}} w - \delta_i z^{\mathrm{T}} z. \quad (14)$$

The following inequality always holds Cao et al. (1998):

$$z^{\mathrm{T}} w \leq \frac{1}{2\delta_i} \| w \|_2^2 + \frac{\delta_i}{2} \| z \|_2^2. \quad (15)$$

From (14) and (15), we have

$$\| z \|_2^2 < \gamma^2 \| w \|_2^2 - \frac{2}{\delta_i} \dot{V}_i(x), \quad (16)$$

where $\gamma = \max_{i \in \mathrm{M}} \sqrt{|1 - 2\delta_i \epsilon_i|}/\delta_i$.

Suppose

$$\{(t_k, i_k) |\ i_k \in \mathrm{M};\ k = 0, 1, 2, \ldots, s;\ 0 = t_0 \leq t_1 \leq t_2 \leq \cdots \leq t_s = T\} \quad (17)$$

is a switching sequence generated by (8) on the interval $[0,\ T]$.

When $t \in [t_k,\ t_{k+1}),\ (k = 0, 1, 2, \ldots, s-1)$, the $i_k$th subsystem of system (9) is activated, from (16), we get

$$\begin{aligned} \int_0^T \| z \|_2^2 \mathrm{d}t &< \int_0^T \gamma^2 \| w \|_2^2 \mathrm{d}t - \sum_{k=0}^{s-1} \frac{2}{\delta_{i_k}} \int_{t_k}^{t_{k+1}} \dot{V}_{i_k}(x) \mathrm{d}t \\ &\leq \int_0^T \gamma^2 \| w \|_2^2 \mathrm{d}t + \frac{2}{\delta} V_{i_0}(x(0)) - \frac{2}{\delta} V_{i_s}(x(T)), \end{aligned} \quad (18)$$

where $\delta = \min_{i \in \mathrm{M}} \{\delta_i\}$. When $x(0) = 0$, $V_{i_0}(0) = 0$. Furthermore, inequality (6) can be established from (18). This completes the proof of Theorem 1. □

***Remark 1*** It is clear that each subsystem exhibits input and output strict passivity in the activation interval, and not in the whole time domain. $H_\infty$ performance is related to $\epsilon_i$ and $\delta_i$, which are the input and output strictly passive indices of each subsystem.

Next, we design the PDSF controller in (5), under the action of the controller, system (1) can be transformed into

$$\begin{cases} \hat{E}_{\sigma(t)} \dot{x} = \hat{A}_{\sigma(t)} x + F_{\sigma(t)} w, \\ z = C_{\sigma(t)} x + G_{\sigma(t)} w, \end{cases} \quad (19)$$

where $\hat{E}_{\sigma(t)} = E_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)e}$, $\hat{A}_{\sigma(t)} = A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)a}$.

Assumption 1 shows that there exists $K_{ie}$ such that $E_i + B_i K_{ie}$ is invertible. Next, we will give a way to construct $K_{ie}$.

From $\mathrm{rank}(E_i) = r_i \leq n$, there exist two nonsingular matrices $M_i$ and $N_i$, such that

$$M_i E_i N_i = \begin{bmatrix} I_{r_i} & 0 \\ 0 & 0 \end{bmatrix}, \quad M_i B_i = \begin{bmatrix} B_{i1} \\ B_{i2} \end{bmatrix}, \quad (20)$$

where $B_{i1} \in \mathrm{R}^{r_i \times p}$, $B_{i2} \in \mathrm{R}^{(n-r_i)\times p}$, and $\mathrm{rank}(B_{i2}) = n - r_i$.

The gain $K_{ie}$ can be constructed as follows

$$K_{ie} = \begin{bmatrix} 0 & B_{i2}^{\mathrm{T}} \end{bmatrix} N_i^{-1}. \tag{21}$$

Obviously, $E_i + B_i K_{ie}$ is invertible.

From (19) and (21), we can obtain

$$\begin{cases} \dot{x} = \hat{E}_{\sigma(t)}^{-1} \hat{A}_{\sigma(t)} x + \hat{E}_{\sigma(t)}^{-1} F_{\sigma(t)} w, \\ z = C_{\sigma(t)} x + G_{\sigma(t)} w. \end{cases} \tag{22}$$

**Theorem 2** *Consider system* (1) *and Assumption 1, if there exist matrices* $X_i > 0$, $W_i$ *and scalars* $\lambda_{ij} < 0$, $(i, j \in \mathrm{M}, i \neq j)$, *such that*

$$\begin{bmatrix} \Omega_{i11} & \hat{E}_i^{-1} F_i - 0.5 X_i C_i^{\mathrm{T}} + \delta_i X_i C_i^{\mathrm{T}} G_i & \delta_i X_i C_i^{\mathrm{T}} \\ * & \epsilon_i I + \delta_i G_i^{\mathrm{T}} G_i - 0.5 G_i - 0.5 G_i^{\mathrm{T}} & 0 \\ * & * & -\delta_i I \end{bmatrix} < 0, \tag{23}$$

*where* $\Omega_{i11} = \hat{E}_i^{-1} A_i X_i + \hat{E}_i^{-1} B_i W_i + (\hat{E}_i^{-1} A_i X_i + \hat{E}_i^{-1} B_i W_i)^{\mathrm{T}} + \sum_{j=1}^{m} \lambda_{ij}(X_i - X_j)$, $K_{ie}$ *is given by (21), and* $K_{ia} = W_i X_i^{-1}$.

*The switching law is designed as*

$$\sigma(t) = \min\{\arg \max_{i \in \mathrm{M}} x^{\mathrm{T}} X_i^{-1} x\}. \tag{24}$$

*Then system* (1) *controlled by* (5) *is QNQS and satisfies* $H_\infty$ *performance* $\gamma = \max_{i\in \mathrm{M}} \sqrt{|1 - 2\delta_i \epsilon_i|}/\delta_i$.

***Proof*** According to Schur complement lemma (see Yu 2002), from (23), we have

$$\begin{bmatrix} \Omega_{i11}^{'} & \hat{E}_i^{-1} F_i - 0.5 X_i C_i^{\mathrm{T}} + \delta_i X_i C_i^{\mathrm{T}} G_i \\ * & \epsilon_i I + \delta_i G_i^{\mathrm{T}} G_i - 0.5 G_i - 0.5 G_i^{\mathrm{T}} \end{bmatrix} < 0, \tag{25}$$

where $\Omega_{i11}^{'} = \hat{E}_i^{-1} A_i X_i + \hat{E}_i^{-1} B_i W_i + (\hat{E}_i^{-1} A_i X_i + \hat{E}_i^{-1} B_i W_i)^{\mathrm{T}} + \delta_i X_i C_i^{\mathrm{T}} C_i X_i + \sum_{j=1}^{m} \lambda_{ij}(X_i - X_j)$.

Substituting $W_i = K_{ia} X_i$ into inequality (25), pre- and post-multiplying the left-hand-side matrix of (25) by $\mathrm{diag}\{X_i^{-1}, I\}$ and its transpose, respectively, and letting $P_i = X_i^{-1}$, we can obtain

$$\begin{bmatrix} \Omega_{i11}^{''} & P_i \hat{E}_i^{-1} F_i - 0.5 C_i^{\mathrm{T}} + \delta_i C_i^{\mathrm{T}} G_i \\ * & \epsilon_i I + \delta_i G_i^{\mathrm{T}} G_i - 0.5 G_i - 0.5 G_i^{\mathrm{T}} \end{bmatrix} < 0, \tag{26}$$

where $\Omega_{i11}^{''} = P_i \hat{E}_i^{-1}(A_i + B_i K_{ia}) + (A_i + B_i K_{ia})^{\mathrm{T}} \hat{E}_i^{-\mathrm{T}} P_i + \delta_i C_i^{\mathrm{T}} C_i + \sum_{j=1}^{m} \lambda_{ij} P_i (P_i^{-1} - P_j^{-1}) P_i$.

Since $P_i > 0$, we get

$$(P_i - P_j) P_j^{-1} (P_i - P_j) \geq 0. \tag{27}$$

From (27) and $\lambda_{ij} < 0$, we have

$$\sum_{j=1}^{m} \lambda_{ij} P_i (P_i^{-1} - P_j^{-1}) P_i \geq \sum_{j=1}^{m} \lambda_{ij} (P_j - P_i). \tag{28}$$

From (26) and (28), we have

$$\begin{bmatrix} \Omega'''_{i11} & P_i\hat{E}_i^{-1}F_i - 0.5C_i^{\mathrm{T}} + \delta_i C_i^{\mathrm{T}} G_i \\ * & \epsilon_i I + \delta_i G_i^{\mathrm{T}} G_i - 0.5G_i - 0.5G_i^{\mathrm{T}} \end{bmatrix} < 0. \tag{29}$$

where $\Omega'''_{i11} = P_i\hat{E}_i^{-1}(A_i + B_i K_{ia}) + (A_i + B_i K_{ia})^{\mathrm{T}} \hat{E}_i^{-\mathrm{T}} P_i + \delta_i C_i^{\mathrm{T}} C_i + \sum_{j=1}^m \lambda_{ij}(P_j - P_i)$.
According to Theorem 1, it can be seen from (24) and (29) that system (1) controlled by (5) is QNQS and satisfies $H_\infty$ performance $\gamma = \max_{i\in\mathrm{M}} \sqrt{|1-2\delta_i\epsilon_i|}/\delta_i$. This completes the proof of Theorem 2. □

***Remark 2*** From the above analysis, it can be seen that the derivative gains of PDSF controllers are designed in advance. Although the step-by-step design brings some conservatism, it greatly reduces the design difficulty of PDSF controllers. In addition, the results of this work are also valid for switched normal systems. Obviously, if the derivative matrices are invertible, the use of PDSF controllers can also promote the system performance.

Next, we study the issue of $H_\infty$ control for system (1) by designing DSF controllers.

$$u_{\sigma(t)} = -K_{\sigma(t)e}\dot{x}. \tag{30}$$

Assumption 1 shows that there exists $K_{\sigma(t)e}$ such that $E_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)e}$ is invertible.
Under the action of (30), system (1) can be transformed into

$$\begin{cases} \dot{x} = \bar{E}_{\sigma(t)}^{-1}A_{\sigma(t)}x + \bar{E}_{\sigma(t)}^{-1}F_{\sigma(t)}w, \\ z = C_{\sigma(t)}x + G_{\sigma(t)}w, \end{cases} \tag{31}$$

where $\bar{E}_{\sigma(t)} = E_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)e}$.

**Theorem 3** *Consider system (1) and Assumption 1, if there exist matrices $X_i > 0$, $W_i$ and scalars $\lambda_{ij} < 0$, $(i, j \in \mathrm{M}, i \neq j)$, such that*

$$\begin{bmatrix} \Xi_{i111} + \Xi_{i112} & \Xi_{i12} & \Xi_{i13} & \Xi_{i14} \\ * & \Xi_{i22} & 0 & 0 \\ * & * & -\delta_i I & 0 \\ * & * & * & (\sum_{j=1}^m \lambda_{ij})X_i \end{bmatrix} < 0, \tag{32}$$

*where* $K_{ie} = W_i X_i^{-1}$, $\Xi_{i22} = \epsilon_i I + \delta_i G_i^{\mathrm{T}} G_i - 0.5G_i - 0.5G_i^{\mathrm{T}}$,

$$\begin{aligned}
\Xi_{i111} &= A_i X_i E_i^{\mathrm{T}} + E_i X_i A_i^{\mathrm{T}} + A_i W_i^{\mathrm{T}} B_i^{\mathrm{T}} + B_i W_i A_i^{\mathrm{T}}, \\
\Xi_{i112} &= (\textstyle\sum_{j=1}^m \lambda_{ij})(E_i X_i + B_i W_i + X_i E_i^{\mathrm{T}} + W_i^{\mathrm{T}} B_i^{\mathrm{T}}) - \textstyle\sum_{j=1}^m \lambda_{ij} X_j, \\
\Xi_{i12} &= F_i + (E_i X_i + B_i W_i)(-0.5C_i^{\mathrm{T}} + \delta_i C_i^{\mathrm{T}} G_i), \\
\Xi_{i13} &= \delta_i E_i X_i C_i^{\mathrm{T}} + \delta_i B_i W_i C_i^{\mathrm{T}}, \\
\Xi_{i14} &= (\textstyle\sum_{j=1}^m \lambda_{ij})(E_i X_i + B_i W_i).
\end{aligned}$$

*The switching law is designed as*

$$\sigma(t) = \min\{\arg\max_{i\in\mathrm{M}} x^{\mathrm{T}} X_i^{-1} x\}. \tag{33}$$

*Then system (1) controlled by (30) is QNQS and satisfies $H_\infty$ performance $\gamma = \max_{i\in\mathrm{M}} \sqrt{|1-2\delta_i\epsilon_i|}/\delta_i$.*

***Proof*** Replacing $E_i^{-1}$ in Theorem 3.1 by $(E_i + B_i K_{ie})^{-1}$ gives

$$\begin{bmatrix} \Phi_{i11} & P_i(E_i + B_i K_{ie})^{-1} F_i - 0.5C_i^{\mathrm{T}} + \delta_i C_i^{\mathrm{T}} G_i \\ * & \epsilon_i I + \delta_i G_i^{\mathrm{T}} G_i - 0.5G_i - 0.5G_i^{\mathrm{T}} \end{bmatrix} < 0, \tag{34}$$

where $\Phi_{i11} = P_i(E_i + B_i K_{ie})^{-1} A_i + A_i^{\mathrm{T}}(E_i + B_i K_{ie})^{-\mathrm{T}} P_i + \delta_i C_i^{\mathrm{T}} C_i + \sum_{j=1}^{m} \lambda_{ij}(P_j - P_i)$.

Pre- and post-multiplying the left-hand-side matrix of (34) by $\mathrm{diag}\{(E_i + B_i K_{ie})P_i^{-1}, I\}$ and its transpose, respectively, letting $P_i^{-1} = X_i$, $W_i = K_{ie} X_i$, and applying Schur complement lemma Yu (2002), we have

$$\begin{bmatrix} \Phi'_{i11} & \Xi_{i12} & \Xi_{i13} & \Xi_{i14} \\ * & \Xi_{i22} & 0 & 0 \\ * & * & -\delta_i I & 0 \\ * & * & * & (\sum\limits_{j=1}^{m} \lambda_{ij}) X_i \end{bmatrix} < 0, \tag{35}$$

where $\Phi'_{i11} = \Xi_{i11} + \sum_{j=1}^{m} \lambda_{ij}(E_i X_i + B_i W_i) X_j^{-1} (E_i X_i + B_i W_i)^{\mathrm{T}}$.

Since $X_i > 0$, the following inequality always holds.

$$[(E_i X_i + B_i W_i) - X_j] X_j^{-1} [(E_i X_i + B_i W_i)^{\mathrm{T}} - X_j] \geq 0. \tag{36}$$

From (36) and $\lambda_{ij} < 0$, we have

$$\sum_{j=1}^{m} \lambda_{ij}(E_i X_i + B_i W_i) X_j^{-1} (E_i X_i + B_i W_i)^{\mathrm{T}} \leq \Xi_{i112}. \tag{37}$$

It is easily seen from (37) that inequality (34) can be deduced from (32). According to Theorem 1, Theorem 3 can guarantee that system (1) controlled by (30) is QNQS and satisfies $H_\infty$ performance $\gamma = \max_{i \in \mathrm{M}} \sqrt{|1 - 2\delta_i \epsilon_i|}/\delta_i$. This completes the proof of Theorem 3. □

***Remark 3*** It is worth pointing out that the design of DSF controllers is much easier than that of PDSF controllers for actual control systems (Ren and Zhang 2010), and the costs of designing and maintaining for DSF controllers are smaller. Therefore, the control strategy should be based on the actual situation to achieve a better control effect.

***Remark 4*** When $\epsilon_i = 0$, Theorems 1, 2 and 3 become the solvable conditions of the $H_\infty$ control problem for switched singular systems based on output strict passivity, and the $H_\infty$ performance indexes can be rewritten as $\gamma = \max_{i \in \mathrm{M}}\{1/\delta_i\}$.

## 4 Numerical examples

In this section, we give the following three examples to show the effectiveness of the proposed methods in this work. The comparison between PDSF and DSF is given in Examples 1 and 2, and the comparison between PDSF and PSF is shown in Example 3.

***Example 1*** Consider a switched singular system in (1) with two subsystems.

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 1 & 4 \\ 2 & -5 & 3 \\ 1 & -3 & 6 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 \\ -1 \\ 1 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \end{bmatrix},$$

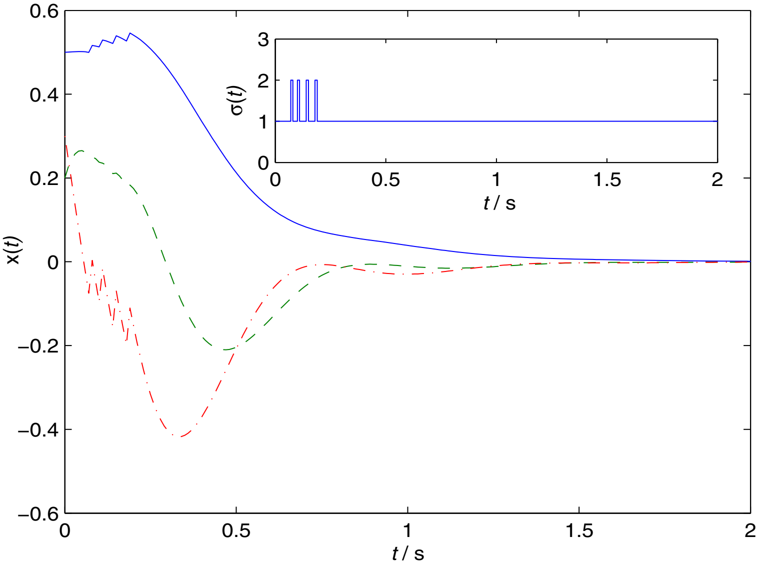


**Fig. 1** State trajectory of the closed-loop system and the switching law based on PDSF

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; A_2 = \begin{bmatrix} 2 & 1 & 5 \\ 0 & -3 & 4 \\ -1 & 2 & -5 \end{bmatrix}, \; B_2 = \begin{bmatrix} 1 \\ 1 \\ -0.5 \end{bmatrix}, \; F_2 = \begin{bmatrix} 0.15 \\ 0.3 \\ -0.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 1 & 0.5 & -1 \end{bmatrix}, \; G_1 = 0.3, \; C_2 = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}, \; G_2 = 0.5.$$

Obviously, $\mathrm{rank}[E_i \;\; B_i] = 3 = n$, $\forall i \in \{1, 2\}$. From (21), we can obtain that $K_{1e} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$, $K_{2e} = \begin{bmatrix} 0 & 0 & -0.5 \end{bmatrix}$.

Letting $\lambda_{12} = \lambda_{21} = -0.5$, $\delta_1 = 0.8$, $\epsilon_1 = 0.1$, $\delta_2 = 0.4$ and $\epsilon_2 = 0.2$, we get $\gamma = 2.2913$. Furthermore, solving inequality (23) in Theorem 2 and from $K_{ia} = W_i X_i^{-1}$, $i \in \{1, 2\}$, we get

$$K_{1a} = \begin{bmatrix} -5.9540 & -6.8035 & -9.4099 \end{bmatrix},$$
$$K_{2a} = \begin{bmatrix} -4.5339 & -4.0009 & -1.9349 \end{bmatrix}.$$

Under the initial condition $x(0) = \begin{bmatrix} 0.5 & 0.2 & 0.3 \end{bmatrix}^{\mathrm{T}}$, when $w = 0$, system (1) controlled by (5) and (24) is QNQS. The state trajectory and the switching law are depicted in Fig. 1.

We construct a function

$$F(z, w) = z^{\mathrm{T}} z - \gamma^2 w^{\mathrm{T}} w \tag{38}$$

and it is a piecewise function because of $\sigma(t)$. When $x(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ and $w(t) = \mathrm{e}^{-0.6t} \sin(3t + 1)$, the curve of $F(z, w)$ is depicted in Fig. 2. Fig. 2 shows that $F(z, w) \le 0$ and $F(z, w) \not\equiv 0$. According to the properties of definite integral, $J_{\mathrm{T}} < 0$ in (6) holds.

Solving inequality (32) in Theorem 3 using the above parameters, we find that the feasible solutions do not exist. We give the following example to analyze the advantages and disadvantages for PDSF and DSF.

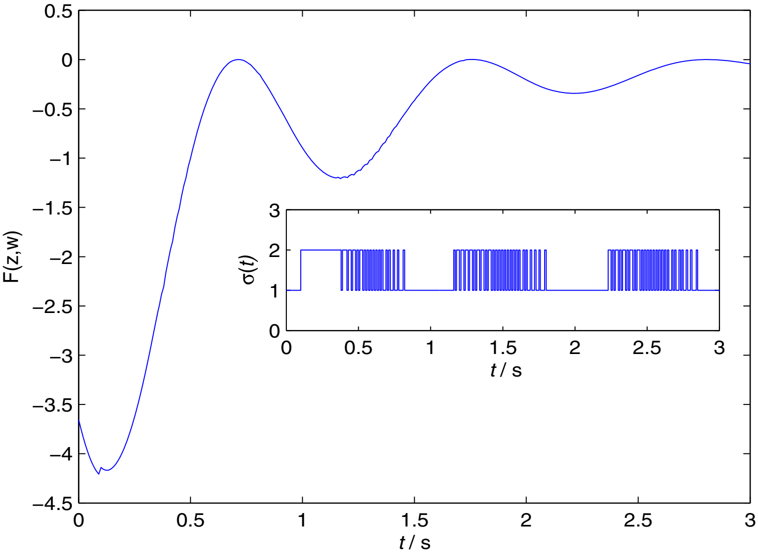


**Fig. 2** The curve of $F(z, w)$ and the switching law based on PDSF

***Example 2*** Consider a switched singular system in (1) with two subsystem.

$$E_1 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} -0.8 & -0.4 & 1.3 \\ 0 & -1.6 & -0.5 \\ 0.3 & 0 & -1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.3 \\ -1 \\ 0.7 \end{bmatrix},\ \ F_1 = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} -1.5 & 0.5 & -0.8 \\ 1 & 1 & 0.5 \\ 0.7 & 0 & -1.5 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.1 \\ 0.5 \\ -1 \end{bmatrix},\ \ F_2 = \begin{bmatrix} 0.15 \\ 0.3 \\ -0.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.2 & 0.5 & 0 \end{bmatrix},\ G_1 = 0.3,\ C_2 = \begin{bmatrix} -0.4 & 0 & 0.2 \end{bmatrix},\ \ G_2 = 0.5.$$

Obviously, $\mathrm{rank}[E_i\ B_i] = 3 = n, \forall i \in \{1, 2\}$.

Letting $\lambda_{12} = \lambda_{21} = -0.5$, $\delta_1 = 0.8$, $\epsilon_1 = 0.1$, $\delta_2 = 0.4$ and $\epsilon_2 = 0.2$, we can obtain $\gamma = 2.2913$. Furthermore, solving inequality (32) in Theorem 3 and from $K_{ie} = W_i X_i^{-1}$, $i \in \{1, 2\}$, we get

$$K_{1e} = \begin{bmatrix} -0.7342 & -2.1290 & 0.2807 \end{bmatrix},$$
$$K_{2e} = \begin{bmatrix} -1.2563 & -4.8410 & -1.7511 \end{bmatrix}.$$

Choose

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, N_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{bmatrix}, M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, N_2 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}.$$

From (21), we can obtain that $K_{1e} = \begin{bmatrix} 0 & 1.4 & 0 \end{bmatrix}$, $K_{2e} = \begin{bmatrix} 0 & -1.6 & 0 \end{bmatrix}$. Furthermore, solving inequality (23) in Theorem 2 and from $K_{ia} = W_i X_i^{-1}$, $i \in \{1, 2\}$, we get

$$K_{1a} = \begin{bmatrix} -2.6598 & -7.6163 & 0.0806 \end{bmatrix},$$
$$K_{2a} = \begin{bmatrix} 1.6104 & 1.9677 & -1.1685 \end{bmatrix}.$$

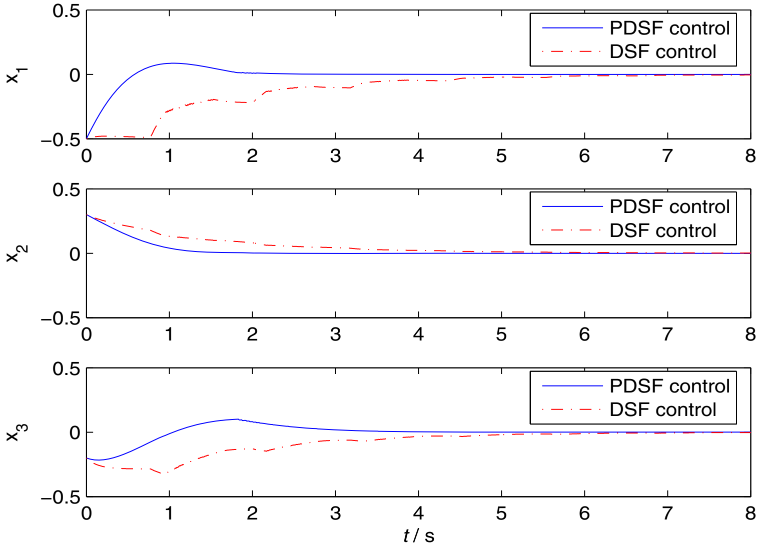


**Fig. 3** State trajectories of the closed-loop system based on PDSF and DSF

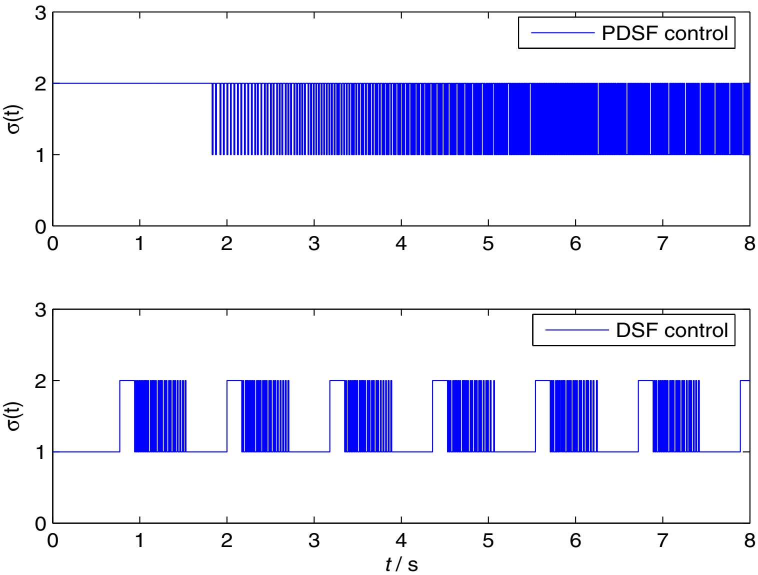


**Fig. 4** Switching laws based on PDSF and DSF

Under the initial condition $x(0) = \begin{bmatrix} -0.5 & 0.3 & -0.2 \end{bmatrix}^{\mathrm{T}}$, when $w = 0$, system (1) controlled by PDSF and DSF, respectively, is QNQS. The stability comparison of the closed-loop systems under these two strategies is shown in Fig. 3, and the comparison of switching laws is shown in Fig. 4.

When $w(t) = \mathrm{e}^{-0.6t} \sin(3t + 1)$, under the zero initial condition, the curves of $F(z, w)$ for system (1) controlled by PDSF and DSF, respectively, is depicted in Fig. 5. It can be

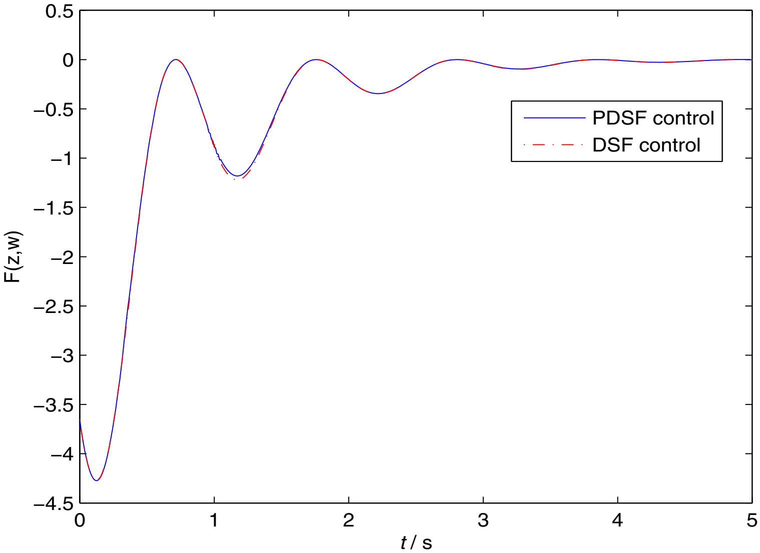


**Fig. 5** The curves of $F(z, w)$ based on PDSF and DSF

seen from Fig. 5 that $F(z, w) \le 0$ and $F(z, w) \not\equiv 0$. According to the properties of definite integral, $J_{\mathrm{T}} < 0$ in (6) holds.

It can be seen from Fig. 3 that both PDSF and DSF control strategies can stabilize system (1). Through the comparison of these two control strategies, it is obvious that PDSF control makes the response faster, the settling time shorter and the dynamic process more stable. Therefore, from the perspective of stability, the effect of PDSF control is better than that of DSF control. It is clear from Fig. 5 that the curves of $F(z, w)$ based on PDSF control and DSF control are almost the same, which shows that the control effect of these two strategies is very close from the perspective of $H_\infty$ performance. Therefore, the choice of control strategies should be based on the actual situation.

***Example 3*** In this example, we introduce a PWM driven boost converter model (see, Example 2 in Zhang et al. (2011)) to show that the proposed methods of this work are also valid for switched normal systems. The boost converter is shown in Fig. 6.

By applying the Kirchhoff laws, we establish the mathematical model of the boost converter as follows:

$$\begin{cases} \dot{u}_C(t) = -\frac{1}{RC}u_C(t) + (1 - s(t))\frac{1}{C}i_L(t), \\ \dot{i}_L(t) = -(1 - s(t))\frac{1}{L}u_C(t) + s(t)\frac{1}{L}u_s(t), \end{cases}$$

which can be further expressed by

$$E_i\dot{x}(t) = A_i x(t), \ i \in \{1, 2\},$$

where $E_1 = E_2 = \mathrm{diag}\{1, 1, 1\}$,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} u_C \\ i_L \\ u_s \end{bmatrix}, \ A_1 = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} & 0 \\ -\frac{1}{L} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ A_2 = \begin{bmatrix} -\frac{1}{RC} & 0 & 0 \\ 0 & 0 & \frac{1}{L} \\ 0 & 0 & 0 \end{bmatrix}.$$

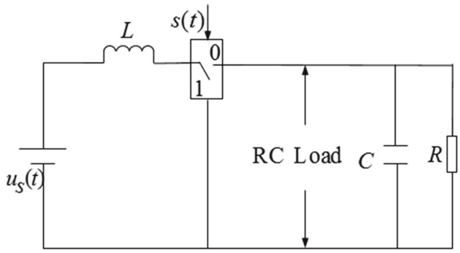


**Fig. 6** Boost converter

For the above switched system, we directly use the parameters in Zhang et al. (2011) as follows:

$$A_1 = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ A_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

and suppose other system matrices to be

$$B_1 = \begin{bmatrix} 1 \\ -0.3 \\ 1 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.6 \\ 1 \\ -0.5 \end{bmatrix}, \ F_1 = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \end{bmatrix}, \ F_2 = \begin{bmatrix} 0.5 \\ 0.3 \\ -0.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -1 & 0.5 & -1 \end{bmatrix}, \ C_2 = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}, \ G_1 = 0.3, \ G_2 = 0.5.$$

We choose $K_{1e} = \begin{bmatrix} 0 & -2 & 1 \end{bmatrix}$ and $K_{2e} = \begin{bmatrix} 0 & 2 & -1 \end{bmatrix}$. Letting $\delta_1 = 0.8, \delta_2 = 0.4$, $\epsilon_1 = 0.2, \epsilon_2 = 0.1$, and $\lambda_{12} = \lambda_{21} = -3$, we get $\gamma = 2.3979$. Furthermore, solving inequality (23) in Theorem 2 and from $K_{ia} = W_i X_i^{-1}$, $i \in \{1, 2\}$, we get

$$K_{1a} = \begin{bmatrix} -70.7821 & 34.7573 & -71.6148 \end{bmatrix},$$
$$K_{2a} = \begin{bmatrix} 148.0341 & -148.3729 & 296.0110 \end{bmatrix}.$$

Moreover, if we choose $K_{1e} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ and $K_{2e} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$. Using the above parameters to solve inequality (23) in Theorem 2 and from $K_{ia} = W_i X_i^{-1}$, we get

$$K_{1a} = \begin{bmatrix} -3.9810 & 1.0859 & -3.5208 \end{bmatrix},$$
$$K_{2a} = \begin{bmatrix} 3.6223 & -4.6014 & 8.5106 \end{bmatrix}.$$

Under the initial condition $x(0) = \begin{bmatrix} -0.3 & 0.4 & 0 \end{bmatrix}^{\mathrm{T}}$, when $w = 0$, system (1) controlled by PDSF and PSF, respectively, is asymptotically stable. The stability comparison of the closed-loop systems under these two strategies is shown in Fig. 7, and the comparison of switching laws can be seen in Fig. 8.

When $w(t) = \mathrm{e}^{-0.6t} \sin(3t + 1)$, under the zero intial condition, the curves of $F(z, w)$ for system (1) controlled by PDSF and PSF, respectively, is depicted in Fig. 9. It can be seen from Fig. 9 that $F(z, w) \leq 0$ and $F(z, w) \not\equiv 0$. According to the properties of definite integral, $J_{\mathrm{T}} < 0$ in (6) holds.

It can be seen from Fig. 7 that both PDSF and PSF control strategies can stabilize the system. Through the comparison of these two control strategies, it is obvious that the PDSF control makes the response faster and the settling time shorter. Therefore, from the perspective of stability, the effect of PDSF control is better than that of PSF control. This shows that the proposed methods of this work are also valid for switched normal systems. Moreover, it can

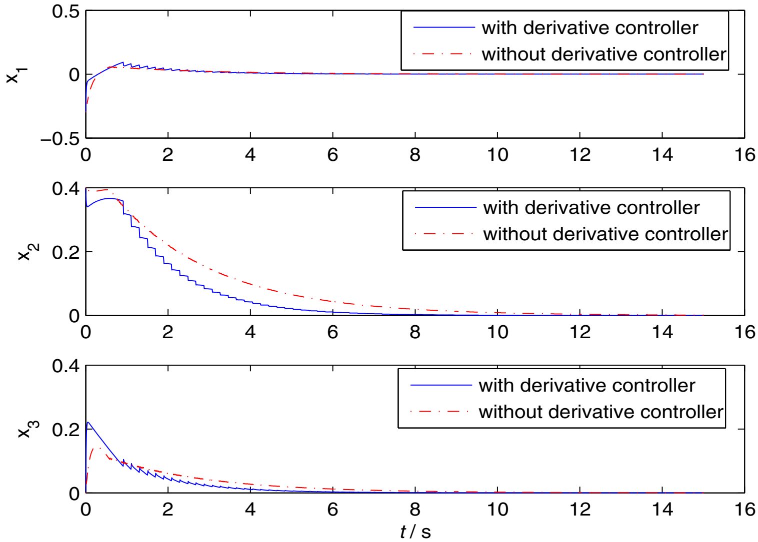


**Fig. 7** State trajectories of boost converter based on PDSF and PSF

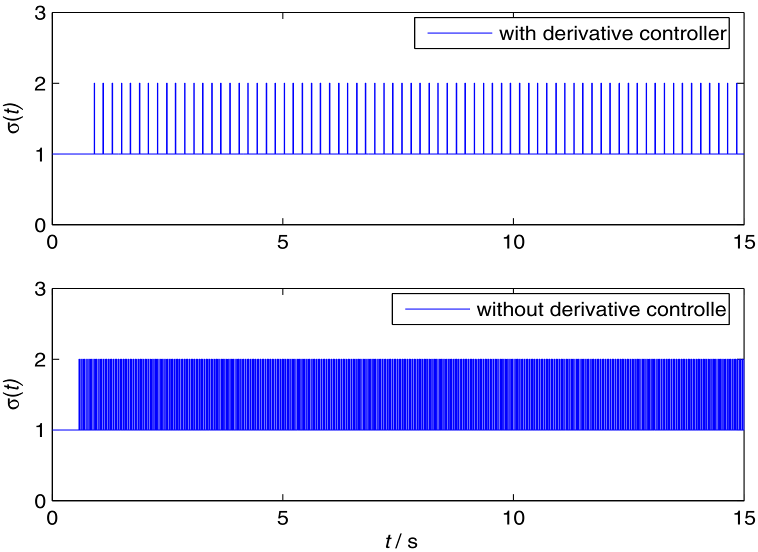


**Fig. 8** Switching laws of boost converter based on PDSF and PSF

be also seen from Fig. 8 that the control effect of PDSF is better than that of PSF. It is clear from Fig. 9 that the curves of $F(z, w)$ based on PDSF control and PSF control are almost the same, which shows that the control effect of these two strategies is very close from the perspective of $H_\infty$ performance. From the above analysis, we conclude that PDSF can be also valid for switched normal systems, and the control effect is related to the selection of derivative gains.

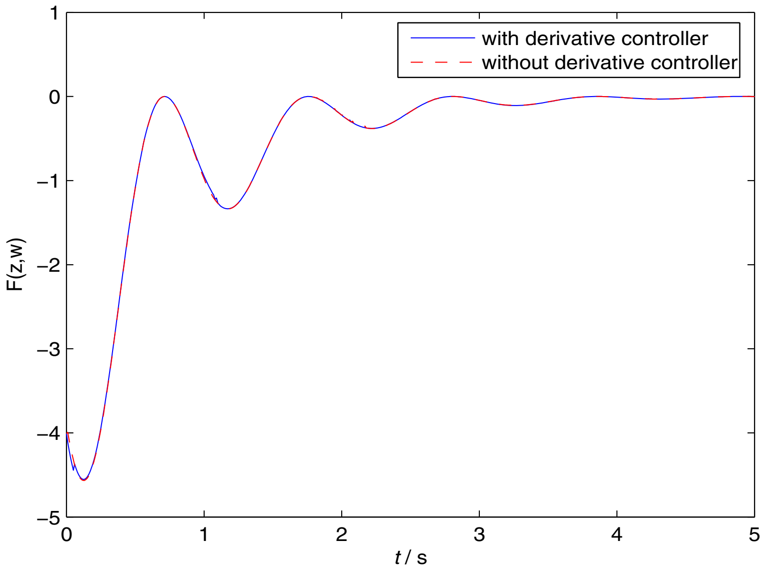


**Fig. 9** The curves of $F(z, w)$ based on PDSF and PSF

## 5 Conclusion

In this work, the issue of $H_\infty$ control based on input and output strict passivity for continuous switched singular systems is studied via PDSF and DSF. The definition of QNQS is extended to switched singular systems. Some sufficient conditions are deduced to guarantee the existence of a solution to the $H_\infty$ control problem. The controller of each subsystem with explicit expression is designed. The results of this work can be easily extended to switched singular systems with time-delays and uncertainties. The derivative gains of PDSF designed in advance may bring some conservatism. How to design the gains of PDSF controllers synchronously is one of our next work.

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